

ACKNOWLEDGMENT

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NOTATION

a	= interfacial area per unit volume of absorber (m^2/m^3)
A^o	= concentration of A in bulk liquid (kmol/m^3)
A^*	= Concentration of A in the liquid at the interface, in equilibrium with gas at interface (kmol/m^3)
B^o	= Concentration of B in bulk liquid (kmol/m^3)
D	= column or tower diameter (m)
\mathcal{D}	= diffusivity (m^2/s)
E	= enhancement factor, dimensionless, $\equiv k_L/k_L^o$
f_g	= friction factor, dimensionless
g	= gravitational acceleration (m/s^2)
G	= average superficial molar gas velocity ($\text{kmol}/\text{m}^2\cdot\text{s}$)
h	= column height (m)
K	= defined in Eq. 11
k_G	= gas-phase mass-transfer coefficient ($\text{kmol}/\text{m}^2\cdot\text{Pa}\cdot\text{s}$)
k_L	= effective liquid-phase mass-transfer (m/s)
k_L^o	= liquid-phase mass transfer coefficient for physical absorption (m/s)
L	= average superficial liquid velocity ($\text{m}^3/\text{m}^2\cdot\text{s}$)
N_A	= average rate of absorption of species A ($\text{kmol}/\text{m}^2\cdot\text{s}$)
P_A	= partial pressure of A in bulk gas phase (Pa)
P_t	= total pressure (Pa)
q	= rate of absorption (kmol/s)
Q_G	= gas flow rate (kmol/s)
Q_L	= liquid flow rate (m^3/s)
Re_G	= gas-phase Reynolds number, $DV_G\rho/\mu$
Re_L	= liquid-phase Reynolds number, $4Q_L\rho_L/\pi D\mu$
Sc	= Schmidt number, $\mu/\rho\mathcal{D}$
t	= exposure time of an element of liquid surface (s)
V_G	= gas superficial velocity (m/s)
V_s	= total liquid inventory (m^3)
y_A	= Mole fraction of A in the gas phase
z	= number of moles of reactant B reacting with each mole of A

α	= solubility coefficient ($\text{kmol}/\text{m}^3\cdot\text{Pa}$)
γ	= expression defined in Eq. 11
δ	= liquid layer thickness on a pipe wall (m)
μ	= viscosity ($\text{Pa}\cdot\text{s}$)
ρ	= density (kg/m^3)

Subscripts and Superscripts

B	= bottom of column
i	= interface composition or partial pressure
I	= region (or bottom of region) in which liquid-phase diffusion controls
II	= region in which gas-phase diffusion controls
LM	= log mean
T	= top of column

LITERATURE CITED

- "Air Pollution and the San Francisco Bay Area," Bay Area Air Pollution District, 11th Ed. (June, 1977).
- Chien, S.-F., and W. Ibele, "Pressure Drop and Liquid Film Thickness of Two-Phase Annular and Annular-Mist Flows," *J. of Heat Trans., Trans. of ASME*, 87, 89 (1964).
- Danckwerts, P. V., *Gas-Liquid Reactions*, McGraw-Hill, 34, 193, 113 (1970).
- Ferguson, P. A., "Hydrogen Sulfide Removal from Gases, Air and Liquids," Noyes Data Corp. (1975).
- Kohl, A. L., and F. C. Riesenfeld, *Gas Purification*, Gulf Publishing, 3rd Ed., 485 (1979).
- Kükner, G., "Alternative Iron Chelating Agents for Oxidative Absorption of H_2 ," M.S. Thesis, University of California Berkeley (1980).
- Lynn, S., and B. Dubs, "Oxidative Removal of Hydrogen Sulfide from Gaseous Streams," U.S. Patent # 4,278,646 (July, 1981).
- Neumann, D. W., "Oxidative Absorption of H_2S and O_2 by Iron-Chelate Solutions," M.S. Thesis, University of California, Berkeley (1981).
- Perry, R. H., and C. H. Chilton, *Chemical Engineers' Handbook*, 5th Ed., McGraw-Hill, (1973).
- Peters, M. S., and K. D. Timmerhaus, *Plant Design and Economics for Chemical Engineers*, 3rd Ed., McGraw-Hill (1980).
- Sherwood, T. K., R. L. Pigford, and C. R. Wilke, *Mass Transfer*, McGraw-Hill, 154, 205 (1975).

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Computer-Aided Synthesis and Design of Plant Utility Systems

A design synthesis procedure is developed for preliminary design of utility systems. Given known steam sources (waste heat and auxiliary boilers) and sinks (heating, process injection, and driver horsepower needs), the algorithm determines the optimal header pressure levels, the distribution of steam turbines in the network, and the steam flows between all devices so as to maximize the real work recovered from the sources. Any number of pressure levels can be accommodated at only modest increase in computational effort.

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SCOPE

With the growing emphasis on energy conservation in process operation and design, there is a need for efficiency in the design of plant utilities systems and in the integration of the utilities system design with the process-side design. Most of the publications dealing with the synthesis and design of utilities systems

have focused on the balance calculations associated with fixed utility system networks with specified header pressure levels. For instance, Gordon et al. (1978) discuss computerized steam balance calculations; Wilkinson (1979) presents a specialized simulation/design approach for performing steam balances and

selecting the operating conditions of steam turbines. Nishio (1977) appears to be the first to consider the problem of selecting optimal header pressure levels and presented a direct search approach coupled with simultaneous solution of the balance equations. More recently, Nishio et al. (1980) proposed the use of loss of available energy as criterion within the framework of the methodology of Nishio (1977). Finally, Grossman and Santibanez (1979) considered the use of mixed integer programming methods to select optimal pressure levels by assigning 0-1 variables to several possible pressure values. This paper discusses the complete problem of synthesizing the utility network: selecting the header conditions and number of pressure levels; selecting the power source and operating conditions for

all process work needs; and balancing the network steam flows. The key assumptions used are the following:

- i. Each heat sink or source involves a single contact with a utility stream.
- ii. The temperature of steam at each level is related to the pressure via an operating curve defined through an efficiency parameter.
- iii. The optimal pressure values will always be one of the minimum/maximum steam pressure levels associated with the waste heat, process heating and process steam needs.

Under these assumptions an efficient synthesis procedure is devised which makes use of dynamic programming and linear programming techniques.

CONCLUSIONS AND SIGNIFICANCE

A synthesis and design procedure for utility systems is developed in terms of two coupled subproblems: header selection and driver allocation. The former subproblem is solved via dynamic programming using as performance index the minimization of the loss of real work. This performance index has the desirable feature of measuring loss relative to a physically attainable rather than a hypothetical reference design. The dynamic programming solution requires an outer iteration loop on the auxiliary boiler load which is proven to converge from a zero initial estimate.

The driver allocation problem is posed as a linear program using as performance measure any linear function of the steam or electricity flows. Exclusion of multistate extraction turbines or avoidance of joint utilization of steam turbines and electric motors to supply a given shaft-work need requires the addition of 0-1 variables and solution of the subproblem as a mixed integer linear program.

Subproblem coupling occurs through the driver efficiencies and auxiliary boiler load. Thus, repeated subproblem solution

cycles are required. Test cases run with a program implementing this methodology show that

- i. Loss of real work decreases with increased number of steam levels. However, beyond six pressure levels the potential for improvement diminishes markedly, for systems operating over the normal pressure ranges used in the process industries.
- ii. The optimal lowest pressure level was always equal to the minimum allowable pressure.
- iii. Assumption (iii) was never violated.
- iv. The upper level superheat temperature is an important design optimization variable.
- v. Excessive staging of extraction turbines or joint steam-electric powering of work needs were only infrequently encountered.
- vi. Convergence of the synthesis procedure was generally quick and always monotonic.
- vii. Multiple process-stream/utility stream contacts can lead to modest improvements in system energy utilization.

INTRODUCTION

A simplified flow diagram of a typical process utilities system is shown in Figure 1. In such a system, steam is generated in boilers at various different pressure levels and is used to satisfy process heating needs, to provide process steam, and to drive steam expansion turbines. Boiler heat sources will either be the combustion of a fuel or the waste heat contained in hot process streams. The fuel-fired boilers will generate steam at the highest pressure level, while the waste heat recovery boilers (1, 2, 3 in Figure 1) will generate steam at pressure levels appropriate for the temperature range of the waste heat source. Process heating requirements (7-10) are differentiated from process steam needs (11-14) in that the saturated condensates from the latter are not returned to the utility system. Returned saturated condensate is normally flashed (4, 5, 6) at the next lowest pressure level to recover saturated steam and a lower-pressure condensate which is passed on to the next level flash. The steam turbines provide plant motive power either directly (15-18) by driving pumps, compressors, as well as other machines, or else indirectly by driving electric generators (19). The generated electricity may, in turn, be used to power electric motors and other devices. In general, back-pressure, extracting, or condensing turbines may be used in both single and multistage configurations. Residual steam at the lowest pressure level is condensed using cooling water, deaerated, combined with make-up boiler feed water, pumped to appropriate pressure levels, and recycled to the boilers.

For purposes of this paper, the design synthesis problem for such

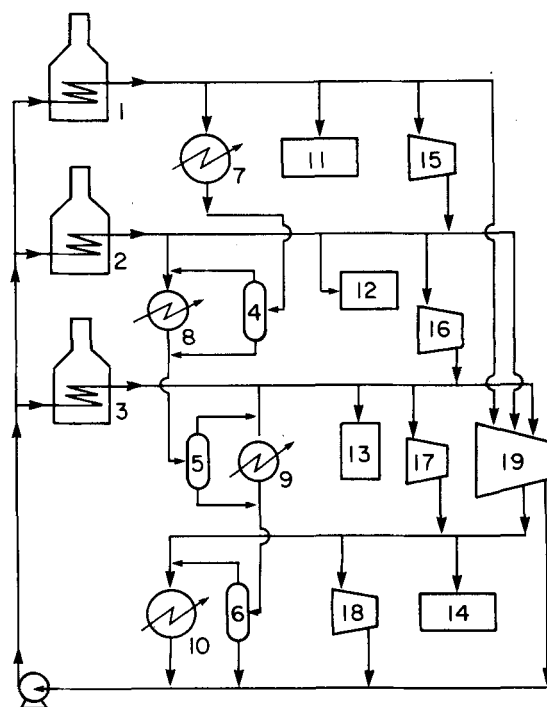


Figure 1. Typical multiple header process utility system.

a system is defined as, given the initial and final temperatures as well as the capacity flow of each waste heat containing stream and each stream requiring heating; the flow and minimum allowable pressure of each process steam need; and the shaft work required of each turbine as well as the electric power needs determine,

- i. The number of steam levels and the temperature and pressure of each level
- ii. The steam level at which each heat and steam need is supplied
- iii. The assignment of steam or electric power sources to each shaft-work need
- iv. The input and output flow and condition of all steam streams in the network
- v. The auxiliary boiler load and condenser cooling water requirements so as to optimize some suitable design criterion

In practice, the design and operation of a utility system is subject to considerable dynamic and stochastic load variations. These effects are not directly considered in the present work but, instead, are assumed to be accommodated by the selection of suitable mean steady-state conditions. Moreover, in view of the definition of utility system heat sources and sinks in terms of streams and their associated inlet and outlet temperatures, our problem can be viewed as containing imbedded within it the classical heat exchanger network synthesis problem (Nishida et al., 1981). To focus attention on the utility side of the design problem, we assume that all feasible and desirable direct stream-to-stream heat exchanges have already been considered and, hence, that the stream heat sources and sinks are only to be contacted with utility streams. Furthermore, we impose the restriction that each stream involves only a single contact with a utility stream, hence, that, as a result of a preanalysis, streams for which multiple contacts are appropriate will have been subdivided into separate streams with their own inlet and outlet temperatures. Finally, boiler feed water pump and boiler combustion air fan power requirements, letdown station and boiler blow-down flows, as well as deaerator steam consumption are not explicitly considered in our analysis to streamline the presentation. These elements can, however, readily be treated as specialized shaftwork and steam or heat needs within the methodology of this paper.

The motivation for the synthesis approach taken in this work stems from the basic difference in the thermodynamic limits associated with heat transfer and those associated with the work produced by steam expansion. Heat transfer is constrained by the need for a finite temperature difference between the hot and cold media. Thus, for each waste heat source and given minimum approach temperature, there is a maximum pressure of steam that can be generated from that waste heat source. Similarly, for each process heating need there is a minimum pressure of steam at which heat transfer can be carried out via steam condensation. If steam is generated considerably below the maximum pressure associated with a given heat source or consumed at a pressure considerably above the minimum pressure required by a heat sink, the heat transfer process becomes thermodynamically inefficient. The heat transfer process is thus rather tightly constrained. On the other hand, a given amount of shaft work can be produced by expanding a suitable steam flow between any two pressure levels. To be sure, the efficiency of the turbine will be influenced by the operating pressures, but this is a secondary effect rather than an operating restriction. As a result of this flexibility of turbines when contrasted to heat transfer devices, the design synthesis can be decomposed into two weakly coupled subproblems: the header selection subproblem and the driver allocation subproblem. These subproblems and their solutions will be presented in detail.

HEADER SELECTION SUBPROBLEM

Assuming single contacts and specific minimum approach temperatures, each heat source can be defined by its total available heat and a maximum steam generation pressure, each sink by the total required heat and minimum acceptable steam supply pressure, and each process steam need by a required mass flow and a minimum steam delivery pressure. Normally a maximum and a

minimum steam pressure will be specified for the utility system design as dictated by steam boiler ratings and cooling water temperatures. Moreover, if an auxiliary fuel-fired boiler is required, it will be operated at the highest pressure level in the system. An auxiliary boiler will be required if the steam generated via waste heat boilers is inadequate to supply all heat and shaftwork needs. Given these conditions, the header selection subproblem reduces to determining the number of steam levels; the temperature and pressure of each level, the auxiliary boiler duty and the condenser load so that the heating, cooling, and total shaft work needs are satisfied while optimizing some suitable performance criterion.

The solution of this problem can be considerably simplified by taking advantage of properties of expansion turbines and by exploiting the stagewise structure of the header network.

Pressure-Temperature Operating Curve

First of all, we observe that if the process steam, shaftwork and electric power generation needs dominate the process heating needs, the primary mechanism for transmitting steam from one pressure level to the next will be through expansion in turbines. When this is the case, the steam states at the various levels can be related through the turbine operating curves.

Recall that the process of expanding a gas through a turbine can be described by a line on an enthalpy-entropy (Mollier) diagram where departure from the vertical (isentropic expansion) is quantified in terms of a turbine (or isentropic) efficiency η . Thus,

$$\eta = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

where h_1 , h_2 , and h_{2s} are, respectively, the enthalpy at the initial state, the actual final state, and the final state of isentropic expansion. Using geometric construction on a Mollier diagram it is easy to show that

$$\tan(\omega) = \frac{1 - \eta}{\eta} \frac{1}{T_{av}}$$

where T_{av} is the average slope of the curve of constant pressure between state (2s) and (2), that is,

$$T_{av} = \frac{h_2 - h_{2s}}{s_2 - s_1}$$

and where ω is the angle between the isentropic and the real expansion operating lines. This equation is the operating line of any given turbine (constant η) over a limited range of operating pressures over which the isobaric lines have approximately parallel slopes. Thus, for a given turbine and specified inlet conditions T_1 , P_1 as well as desired outlet pressure P_2 , there exists a mechanism for calculating the outlet temperature T_2 .

Clearly the steam states at the various levels in the header network can be related via operating equations of the above type. Thus, if the turbine efficiencies (or a single average efficiency) are specified, given the state at one level (typically that of the highest pressure steam), the states of all other levels can each be defined in terms of a single variable. For instance, if the pressure levels are specified, the temperatures will be fixed via the operating equations. Note that if an average value of η is used, since the slopes of the isobaric lines do shift somewhat, the locus of operating points will actually form a convex curve, Figure 2. Furthermore, for given initial pressure this locus will have a significant dependence on both the initial temperature and η . For instance, in Figure 2 curve OA has initial temperature 811 K and efficiency 0.6; curve OB has efficiency 0.7; while, curve MC has initial temperature 783 K and efficiency 0.6.

In summary, since the steam states, at all levels except the first, will be related via a convex operating curve, the dimensionality of the header selection subproblem is reduced from two variables per level (T and P) to a single variable for each level, the pressure being the most convenient. However, the temperature at the highest pressure level should be retained as an important optimi-

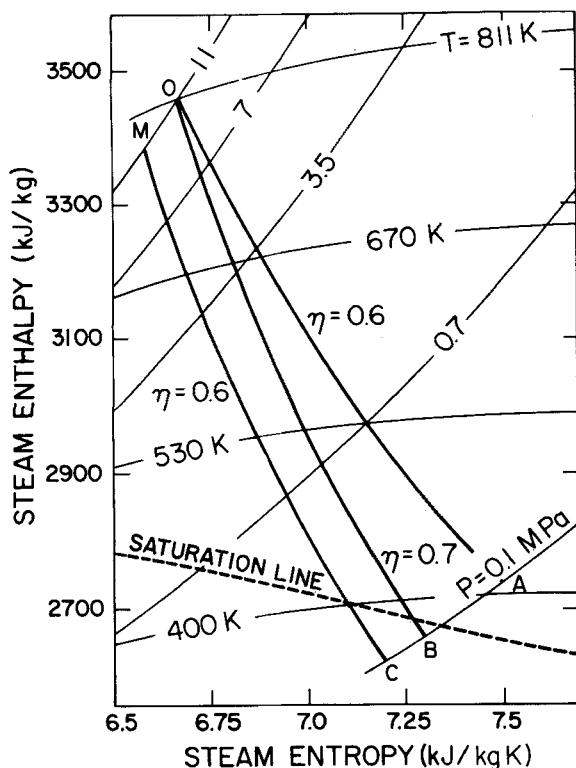


Figure 2. Steam system operating curve.

zation variable because of its strong influence on the operating locus, while the turbine efficiency should be treated as a key design parameter.

Performance Criterion

An economic criterion that takes into account the network capital and operating costs would normally be the choice in this or most other design studies. In the present instance use of such a criterion proves impractical because of difficulties in evaluating the major cost contributions. For instance, one of the major cost items in a utility system is the piping cost of the header network. This cost is clearly a function of the selected steam pressure levels and flows, but the evaluation requires knowledge of the length of the piping as well as of the number of fittings, unions, etc. that are employed. Unfortunately such information is not available until an actual plant layout is defined. Similarly the cost of waste heat boilers, heat exchangers, as well as steam turbines will be functions not only of the heat/work duties but also of the steam flows, temperatures and pressures. Accurate calculations of such costs require a level of detail far in excess of that appropriate for a preliminary design. In view of these complexities and because of the growing importance of energy conservation, an energy-based criterion was selected for the header selection subproblem.

Recall from the subproblem definition that the source and sink heat transfer rates and the shaftwork and electricity needs are fixed. Consequently, from an overall energy balance the difference between the auxiliary boiler heat load and the condenser duty is fixed. Selection of the number of steam levels and their pressures will primarily influence the amount of work that can be extracted from the network or, alternatively the quantity of energy that is degraded in transferring heat from fuel and waste heat sources to process steam and heat sinks as well as in performing work to satisfy process power needs. For the last several years, various second law criteria typically, involving the availability function ψ , have been proposed for evaluating such energy losses (for example, deNevers and Seader, 1979). For a system operating between two states, the difference between the availabilities in these states is equal to the maximum reversible work that can be performed during this change of state (van Wylen, 1967). The disadvantage in using the

availability function as a performance criterion is that it involves a reversible work increment which can normally not be achieved in any real process since it would require infinite heat exchangers, Carnot cycles, and other purely conceptual devices.

In the present application, a more satisfactory criterion can be defined which measures the loss of real work of any selected network relative to a physically implementable but generally impractical reference network. For a utility system, such a reference network will be one in which as many pressure levels are selected as there are waste heat sources, heat sinks and specified steam needs. In this system each waste heat boiler will be permitted to generate steam at its maximum allowable pressure and each heat sink and steam need will be supplied steam at its minimum acceptable pressure level. By successively expanding the excess steam at each pressure level to the next lower pressure level through real turbines until the lowest pressure level is reached, the maximum real work extractable from the given sources will be obtained. Although the number of pressure levels in this reference network will be finite, it will normally be much too large for practical plant operation. Instead, fewer pressure levels will usually be selected, thus, resulting in a loss of real work.

The performance criterion for the system will thus consist of the following three loss terms. In each case, the loss of real work will be the difference between the real work when P is selected as pressure level and the real work that could have been achieved if the pressure P_i characteristic of source or sink i had been selected.

Steam Generation. For each waste heat source i , $i = 1, \dots, I$, let P_i be the maximum steam generation pressure, Q_i the available heat, and P_L the pressure at the lowest level. If, for source i , steam is generated at some pressure P with $P \leq P_i$, it is easily seen that the loss of real work is equal to

$$\Delta W_i(P) = Q_i(HS(P_L) - HW(P_L)) \left[\frac{1}{HS(P) - HW(P_L)} - \frac{1}{HS(P_i) - HW(P_L)} \right] \quad (1)$$

Steam Condensation. For each heat sink j , $j = 1, \dots, J$, let P_j be the minimum allowable steam delivery pressure and Q_j be the required heat. Assume that steam superheat will be removed by the addition of saturated water and that the saturated condensate is returned to the system. If steam is supplied at pressure P with $P \geq P_j$, the loss of real work is given by,

$$\Delta W_j(P) = Q_j \left[\frac{HS(P) - HS(P_L)}{HS(P) - HW(P)} - \frac{HS(P_j) - HS(P_L)}{HS(P_j) - HW(P_j)} \right] \quad (2)$$

Process Steam. For each process steam need k , $k = 1, \dots, K$, let P_k be the minimum allowable delivery pressure and M_k be the required mass flow rate. Assuming no condensate is returned to the system, if steam is supplied at $P \geq P_k$, the loss of real work will be equal to,

$$\Delta W_k(P) = M_k[HS(P) - HS(P_k)] \quad (3)$$

Given any L pressure levels P_l , $l = 1, \dots, L$ such that

$$P_{\max} \geq P_1 > P_2 > \dots > P_l > \dots > P_L \geq P_{\min} \quad (4)$$

we can associate with each level l the index sets,

$$\alpha_l = \{i: P_{l-1} > P_i \geq P_l, i = 1, \dots, I\}$$

$$\beta_l = \{j: P_l \geq P_j > P_{l+1}, j = 1, \dots, J\}$$

$$\gamma_l = \{k: P_l \geq P_k > P_{l+1}, k = 1, \dots, K\}$$

The total loss associated with the L pressure level network with pressures P_l as given above, will thus be

$$\sum_{l=1}^L \left\{ \sum_{i \in \alpha_l} \Delta W_i(P_l) + \sum_{j \in \beta_l} \Delta W_j(P_l) + \sum_{k \in \gamma_l} \Delta W_k(P_l) \right\} \quad (5)$$

Implicit to this formulation is the fact that each P_l will, through the operating curve, have associated with it a temperature T_l .

In addition to these dependent variables, the above loss function also involves two implicit independent variables, namely, the

temperature T_{\max} selected for the maximum pressure and the heat load Q_b supplied via an auxiliary boiler.

Note that since the operating curve incorporates a turbine efficiency, the enthalpy differences evaluated at these conditions will correspond to real rather than isentropic work.

Constraints

In addition to satisfying inequalities (Eq. 4), the pressure levels P_l , boiler load Q_b , and maximum superheat temperature T_{\max} must be selected so that the total shaft work need of the system is satisfied. The total shaft work need will be satisfied by expanding all steam in excess of the direct plant needs through turbines. From a material balance at each level l , the excess steam flow S_l at level l will be given by,

$$S_l = \sum_{i \in \alpha_l} \frac{Q_i}{HS(P_l) - HW(P_l)} - \sum_{j \in \beta_l} \left(\frac{Q_j}{HS(P_l) - HW(P_l)} \right) - \sum_{k \in \gamma_l} M_k \quad (6)$$

This equation can be corrected for condensate flash. For simplicity, these additional terms are not shown.

If W_n , $n = 1, \dots, N$ are the N shaftwork needs of the system, then the work constraint is,

$$\sum_{l=1}^L S_l (HS(P_l) - HS(P_L)) - \frac{1}{\eta_m} \sum_{n=1}^N W_n = 0 \quad (7)$$

where η_m is some averaged mechanical efficiency whose value will depend upon the solution of the driver allocation subproblem.

Dynamic Programming Solution

For fixed L , T_{\max} , and Q_b , the objective function (Eq. 5) and constraints (Eqs. 4, 6 and 7) and the operating curve,

$$T_l = c(T_{\max}, P_{\max}, \eta) \quad (8)$$

are all separable, providing that P_L , the lowest pressure level value is known. Consequently, the minimization of Eq. 5 can be carried out using a dynamic programming procedure (Bellman, 1967). Each stage of the serial system will represent one level; the stage decision variables will be the pressures, P_l ; the state variables will be T_l , S_l , and P_l ; Eqs. 6 and 8 will serve as state transformations; and the stage return function for stage l will be given by the quantity in brackets in Eq. 5.

Following the principle of optimality, solution will begin with the first stage (highest pressure). For each value of P_2 , P_1 is selected so as to minimize the stage 1 return function. Then for all values of the third pressure level, the combined optimum over stages 1 and 2 is found and so on until the lowest pressure level is reached. The possible need for an outer iteration loop to adjust the nonseparable variable P_L is obviated by the following proposition.

Proposition 1. The objective function (Eq. 5) is monotonically increasing in P_L . For proof, see Appendix A.

Since Eq. 5 is monotonically increasing in P_L , the optimum value of the lowest pressure level will always be equal to the lower bound P_{\min} . Consequently, P_L can be set to P_{\min} in advance and an outer loop is not required.

The remaining three potential optimization variables not directly accommodated by the dynamic programming procedure are L , T_{\max} , and Q_b . The first two also affect the driver allocation subproblem and thus must be treated via an upper level search once both the network selection and the driver allocation subproblems have been solved. The boiler load Q_b , however, can be treated at the subproblem level by simply repeating the solution of the dynamic programming problem using the successive Q_b estimates to define the load of an equivalent waste heat boiler at pressure P_{\max} . Estimates of Q_b are updated as follows.

Q_b Adjustment

1. Set $Q_b = 0$ and determine the optimal P_l and S_l , $l = 1, \dots, L$.
2. If Eq. 7 takes on a value ≥ 0 , no auxiliary boiler is required. $Q_b = 0$ is optimal.

3. Calculate S_l so that Eq. 7 is satisfied and then use Eq. 6 to determine the corresponding Q_b .

4. Solve the D.P. subproblem.

5. If Eq. 7 is sufficiently close to zero, terminate. The current Q_b is optimal. Otherwise, go to Step 3.

Proposition 2. The Q_b Adjustment Procedure converges. For proof, see Appendix B.

Discretization

While the header selection subproblem is posed with the pressure levels P_l as continuous variables, implementation of the dynamic programming procedure requires that each of these variables be discretized. Specifically, the theoretical requirement, that at state l for each value of the upstream pressure P_{l-1} , the optimum value of P_l and the corresponding loss function value be determined and stored, would typically be satisfied by subdividing the range of P_{l-1} into a sufficiently fine grid, solving and storing optimal values at these grid points, and interpolating between these grid points if intermediate values prove necessary subsequently. In the present application a natural discretization is however introduced by using as grid points the pressures P_j , $j = 1, \dots, J$, and P_k , $k = 1, \dots, K$. For convenience, we will refer to the set obtained by combining and ordering these pressure values as the *candidate set*. While it can not be proven that the optimal pressure values will always be members of the candidate set, it is possible to show (Appendix C) that the likelihood of optimal pressure values lying between the candidate pressures is quite low. In test problems that we have run, the optimum pressures have always been confined to the candidate set. Thus, based on our experience, a safe and reasonable approach would be to use the candidate set and to supplement it with additional pressure values inserted to subdivide any larger intervals between the candidate points.

DRIVER SELECTION SUBPROBLEM

Upon solution of the header selection subproblem, for fixed L and T_{\max} , the optimal P_l , T_l , S_l , $l = 1, \dots, L$ as well as boiler load Q_b will have been determined. In the driver selection subproblem, the excess steam flows S_l available at each pressure level are allocated among the shaftwork needs W_n and (if appropriate) to an electric power generator (W_{n+1}) so as to satisfy these work needs. In principle, the work needs can be met either from steam turbines or by electric motors for which the power is obtained from the generator.

Let the variables F_{lkn} denote the flow of steam from level l to level k for turbine n ($k > l$) and E_n denote the electricity usage of work need n . Furthermore, let the variables E_{ad} and S_{ad} denote the import of electricity or steam, where for simplicity we assume import steam is supplied at the highest pressure level. Treatment of steam import at multiple pressure level requires only minor modification of the subproblem constraints.

The constraints defining the driver selection subproblem consist of the energy balances on each turbine (work need), the generator energy balance, and the mass balances at each pressure level.

1. Energy Balance for Each Turbine

$$\sum_{l=1}^{L-1} \sum_{k=l+1}^L F_{lkn} (H_l - H_k) \eta_{lkn} + E_n \eta_n \geq W_n, n = 1, \dots, N \quad (9)$$

Note that the work need can be satisfied either by steam expansion or by electric power.

2. Energy Balance for the Generator

$$\sum_{l=1}^{L-1} \sum_{k=l+1}^L F_{lK,N+1} (H_l - H_K) \eta_{lK,N+1} + E_{ad} \geq \sum_{n=1}^N E_n + E \quad (10)$$

3. Steam Balances at Each Pressure Level

At the highest pressure level,

$$S_1 + S_{ad} - \sum_{n=1}^{N+1} \sum_{k=2}^L F_{lkn} = 0$$

At all other levels,

$$S_k + \sum_{n=1}^{N+1} \sum_{l=1}^{k-1} F_{lkn} - \sum_{n=1}^{N+1} \sum_{j=k+1}^L F_{kjn} = 0 \quad k = 2, \dots, L-1 \quad (11)$$

Of course, both the F_{lkn} and E_n variables must be nonnegative.

For a system comprised of L steam levels and N drivers, the above set of three constraint types will involve $(N+1)L(L-1)/2$ steam flow variables as well as $N+1$ electricity rates. The total constraint set will consist of $L-1$ equalities and $(N+1)$ inequalities.

A typical performance function for the driver selection subproblem could be the cost of additional energy inputs to the plant. Thus, the flows would be allocated so as to,

$$\text{Minimize } z = C_1 E_{ad} + C_2 S_{ad} \quad (12)$$

By appropriate choice of C_1 and C_2 one could either exclude one type of input or else obtain an economic balance between steam (fuel) and electricity inputs. Alternatively, one could seek to minimize the condensate recycle or the cooling water requirements or any other linear function of the model variables. Criterion 12 was used in the present work.

Solution Approaches

Minimization of Eq. 12 subject to constraints (Eqs. 9–11) is a standard linear programming problem which can be solved using conventional methods. However, a purely linear programming solution can admit two physically unsatisfactory situations: satisfaction of a work need using both steam and electricity and utilization of an excessive number of steam levels in a turbine. Clearly a given work need should be supplied either via an electric motor or using a steam turbine. Moreover, it may be desirable to limit the number of extraction stages for turbines, particularly for small capacity applications. These types of limitations can be enforced in two ways: through additional constraints or through modification of the objective function.

In the former case one can introduce a 0–1 variable corresponding to the total steam flow through each driver and a 0–1 variable corresponding to the electric flow to each driver. Then joint electricity and steam utilization in a device will be disallowed by requiring that the sum of the two corresponding 0–1 variables be less than or equal to one. Similarly one can introduce a 0–1 variable for each steam flow to indicate whether or not that flow is non-zero. Then, the desired maximum number of feed or extraction stages (M) can be controlled by imposing a constraint requiring that the sum of the 0–1 variables associated with a driver be less than or equal to M . These modifications convert the continuous linear program to a mixed integer LP (MIP). Although commercial mathematical programming software can routinely solve such problems, the branch and bound based procedures typically employed make solution more expensive.

Alternatively one can achieve the same results by inserting penalty terms into the objective function. For instance, by inserting a quadratic term,

$$\Theta \left(\sum_l \sum_k F_{lkn} \right) E_n$$

where Θ is a sufficiently large number, the requirement that work need n be satisfied either with electricity or by steam turbine will be met during the course of minimizing the objective function. In this case, the LP is converted to a quadratic program for which efficient solution methods are also available. Although the dimensionality of the problem is not increased, the possible need for adjustments of parameter Θ is introduced.

Since the LP formulation involves only $L+N$ constraints, at most $L+N$ of the roughly $NL^2/2$ variables will be nonzero at the

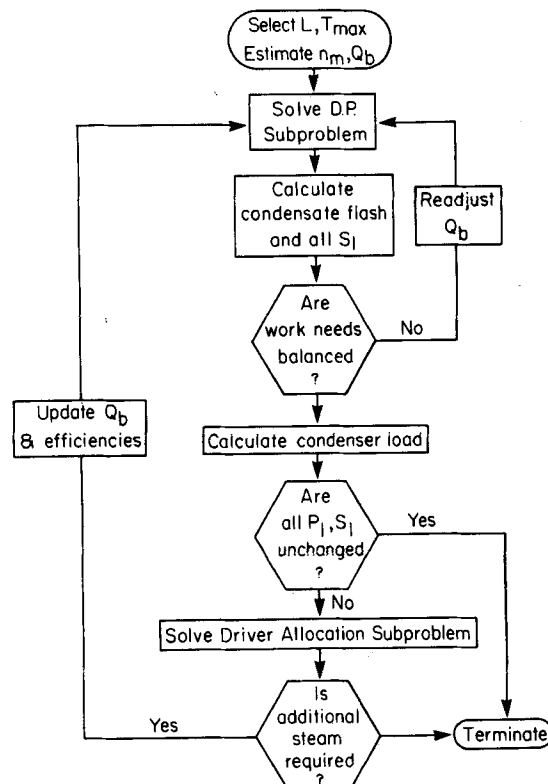


Figure 3. Iteration scheme for synthesis procedure.

LP optimum. Furthermore, for most applications N is considerably larger than L and, because of constraints (Eq. 9), at least one nonzero variable must occur for each of the N work needs. Thus, the likelihood of multiple nonzero variables for any given work need is small. In our experience the preferred approach to the above two potential solution difficulties is to always solve the continuous LP formulation first. If excessive staging or joint steam and electricity supply should arise in the resulting solution, 0–1 variables and their associated constraints should be imposed only for the offending work needs and the problem should be resolved as a MIP.

SUBPROBLEM COUPLING

As described in the preceding, for fixed L and T_{max} , the header selection subproblem will involve repeated dynamic programming solutions with iteration on the required auxiliary boiler heat input. The solution obtained will, however, depend upon assumed values of the driver mechanical efficiencies, the electric generator efficiencies, and total electric power needs. These values can be updated only once the driver selection subproblem is solved. Moreover, as a result of changes in the electricity requirements and efficiencies, additional auxiliary boiler duty may be required. These variables thus serve to couple the two subproblems and impose the need for repeated solution passes through both subproblems as shown in Figure 3. Convergence of the coupling iterations is quite rapid and can be proven using arguments analogous to those given in Appendix B for the Q_b adjustment procedure.

The overall strategy outlined above treats L , the number of pressure levels and T_{max} , the auxiliary boiler superheat temperature as fixed. Optimization with respect to the number of levels is easily carried out by repeated execution of the synthesis procedure for different values of L . Since normal practice is to use between two and seven pressure levels, the additional computational burden is minimal.

Note while the number of heat transfer contacts is independent of L , as L increases more of the contacts are likely to reach minimum approach temperature limits. Consequently, as L is increased

TABLE 1. SPECIFICATIONS FOR EXAMPLE

Max. Steam Pres.:	11.15 MPa	ΔT_{\min} (Boilers) 16.7, K
Max. Steam Pres.:	811 K	ΔT_{\min} (Heaters) 11.1 K
Min. Steam Pres.:	101 kPa	Electricity Needs 1×10^5 kW
Average Turbine Efficiency:	0.70	
Electric Motor Efficiency	0.85	

No.	Inlet Temp. (K)	Outlet Temp. (K)	Heat Capacity Flow 10^4 kJ/K-h
<i>Available Hot Streams</i>			
1	755	505	9.6
2	783	505	52.3
3	655	561	47.3
4	616	533	56.9
5	811	538	11.7
6	866	533	3.8
<i>Heating Needs</i>			
1	361	516	1.9
2	300	383	6.7
3	323	373	1.8
4	373	544	1.8
5	344	423	3.8
6	423	428	95.0
7	316	523	1.9
8	361	494	2.8
<i>Process Steam Needs</i>		<i>Shaftwork Requirements</i>	
No.	Pres. (MPa)	Steam Needed (kg/h)	No. (kW)
1	8.27	2,270	1 31,065
2	6.55	295	2 6,338
3	2.07	20,400	3 2,760
4	3.45	50	4 6,860
5	4.82	340	5 3,730
			410

the heat transfer surface requirements will increase. The optimum L must, therefore, be selected based on a cost criterion and will occur at the point of balance between the incremental heat transfer surfaces as well as headers and the additional savings in fuel or imported electricity. The selection of T_{\max} is best achieved by "tuning" the design once the number of pressure levels has been established, since the allowable T_{\max} is governed by boiler codes and manufacturer specifications. Tuning can be achieved by re-running several cases.

RESULTS

A program implementing our design synthesis methodology has been developed and is available from the authors. The program employs the steam property routines from PPROPS (Overturf, 1979), estimates turbine and mechanical efficiencies using the methods of Spencer et al. (1962), and solves the LP subproblem using a library routine. Execution of a problem of the size defined by the data of Table 1 with six pressure levels, required about 100 s on the Purdue CDC6500 and 130 K octal words of memory to load.

For illustrative purposes, consider the design of a network for the system defined by Table 1. The system has six waste heat sources, eight heating requirements, five defined process steam needs, and six shaft work needs plus electric power needs. The optimal five-level design is summarized in Table 2. Because of the electric power needs, a considerable fuel fired boiler load is imposed. All other work needs are supplied by expansion turbines and, in most cases, these turbines are simple single inlet pressure/single outlet pressure devices. The first work need is supplied with steam at three pressure levels and exhausts at two pressure levels. If this type of operation is deemed unacceptable, then the driver alloca-

TABLE 2. RESULTS FOR NETWORK WITH 5 LEVELS

Steam Level Properties

Level	Pres. (MPa)	Temp. (K)	Net Flows (10^3 kg/h)
1	11.15	811	689.3
2	8.00	767	52.76
3	2.76	638	4.67
4	1.93	600	11.31
5	0.101	367	—

Steam Level Assignments

Level	Hot Stream No.	Heating Need No.	Process Steam Need No.
1	6	—	1
2	2,5	1,4,7,8	2,4,5
3	1,3	—	3
4	4	2,3,4,5	—
5	—	—	—

Driver Allocation

(Steam Flows, 10^3 kg/h)

Work Need	Pressure Level					Electricity
	1	2	3	4	5	
1	80.61 (in)	85.18 (in)	4.67 (in)	4.67 (out)	165.8 (out)	—
2	—	—	—	59.44 (in)	59.44 (out)	—
3	—	—	—	26.91 (in)	26.91 (out)	—
4	—	—	—	64.19 (in)	64.19 (out)	—
5	—	—	—	35.71 (in)	35.71 (out)	—
6	32.42 (in)	32.42 (out)	—	—	—	—
Generator	576.3 (in)	—	—	170.3 (out)	406.0 (out)	10^5 kW

External Requirements

Fuel-Fired Boiler Load: 2.086×10^9 kJ/h

Cooling Water Need: 6.456×10^6 kg/h

TABLE 3. CHANGE OF STEAM LEVELS

No. of Steam Levels	Work (kW)	Network, Max. Attainable, %	Improvement from Previous Network, %
<i>Case A: Temperature at 11.15 MPa, 811 K</i>			
3	8,149.0	55.9	—
4	11,573.0	79.3	42.0
5	12,856.0	88.1	11.1
6	13,337.0	91.4	3.7
7	13,593.0	93.2	1.9
<i>Case B: Temperature at 11.15 MPa, 783 K</i>			
3	10,117.0	77.2	—
4	11,636.0	88.8	15.0
5	12,338.0	94.2	6.0
6	12,682.0	96.8	2.8

TABLE 4. CHANGE OF GENERATED WORK WITH VALUE OF STEAM TEMPERATURE AT HIGHEST PRESSURE (11.15 MPa)

Temp. (K)	Generated Work (kW)	
	Case A	Case B
811	12,856.0	12,185.0
797	13,024.0	12,749.0
783	13,962.0	12,338.0
769	14,318.0	—
755	14,554.0	13,071.0
727	14,774.0	13,679.0
700	14,683.0	13,676.0
686	14,605.0	13,475.0
672	14,474.0	—

tion problem would be resolved with the addition of 0–1 model elements involving work need 1. The selection of steam or electricity as motive source is of course strongly influenced by the assumed electric motor efficiencies. If the efficiency is increased from 0.85 to 0.95, then in the resulting design (results not shown) four of the six work needs do become electrically supplied.

As shown in Table 3, for two different cases, the work generated by a network increases monotonically as the number of pressure levels is increased. However, when compared to the maximum amount of real work that could be generated if a pressure level had been included for each source or sink (a total of 15 in these examples) the potential for improvement diminishes beyond six pressure levels.

The effects of other design variables can also be examined via case studies:

Superheat Temperature T_{max} . As shown in Table 4 for two different networks, the choice of T_{max} has a significant effect (as large as 15%) on the work that can be generated from a network. Note that the effect is not monotonic; rather, for a given network there appears to be a value of T_{max} which yields the maximum work. Recall that T_{max} is one of the two parameters which fix the system operating curve and thus the choice of T_{max} actually influences the temperatures of all steam level. Decreasing T_{max} closer to saturation, lowers the operating temperatures. For cases in which the temperature profile pinch point occurs at the superheated steam end, a lower T_{max} will thus allow generation of steam from the waste heat sources at higher pressures. Generation at higher pressures increases the work which can be recovered by expansion of steam. On the other hand with a higher T_{max} , steam generated at the highest pressure will have a larger enthalpy value, hence, will yield more work when expanded to the lowest pressure. These counteracting tendencies essentially serve to define the maxima which are observed.

Average Turbine Efficiency and Minimum Approach ΔT . The average turbine efficiency defines the operating curve and thus affects the total design. The effect is monotonic: decreased efficiency decreases the recovered work. For the system of Table 1 a change in efficiency from 75 to 65% reduces the generated work

by about 10%. Similarly, the minimum exchanger ΔT 's limit the pressures at which steam can be generated or condensed. Increasing the ΔT 's caused increased loss of driving force, hence, monotonically decreases the generated work. However, the effect of ΔT is much less significant. For instance, in the example an increase in the boiler and heater ΔT_{min} to 27.8 and 16.7 K respectively reduces the generated work by 1.6%. Further increases to 38.9 and 27.8 K, respectively, causes 3.2% reduction.

Multiple Stream-Utility Contacts. The methodology developed here assumes that each stream will involve only a single contact with steam. If the pinch point for heat transfer occurs at the point at which saturated liquid begins to boil then clearly increased efficiency can be attained by dividing the stream into two segments at the pinch point temperature. Each segment can then be used to generate steam at different pressures. Potential improvements of 3 to 4% in the work generated have been attained through splitting in cases considered in this study.

ACKNOWLEDGMENT

This work was carried out under support provided by Conoco, Inc. and the Purdue Research Foundation. C. M. Lin made important improvements to the computer code and developed some of the numerical results which were reported.

NOTATION

$c()$	= steam system operating curve
E_m	= flow of electrical energy at work need n
E	= specified electricity need of the plant
F_{lkn}	= flow of steam of pressure P_l through turbine n to pressure level P_k
$g()$	= operator denoting total system energy balance
h	= enthalpy per unit mass
H_l	= enthalpy of steam of pressure P_l
$HS(P)$	= enthalpy of superheated steam of pressure P per unit mass
$HW(P)$	= enthalpy of saturated water of pressure P per unit mass
I	= number of waste heat sources
J	= number of heat sinks
K	= number of process steam needs
L	= number of steam levels
$m()$	= mapping which yields the optimal system pressure levels for given auxiliary boiler load
N	= number of work needs
P	= pressure
P_L	= pressure of lowest steam level of the plant
Q	= heat load of steam-generating boiler, waste heat boiler, or heat sink
s	= entropy per unit mass
S_{ad}	= additional steam need of the plant
S_l	= net steam available at level l
M_k	= process steam flow for need k
T	= absolute temperature
W_n	= work requirement of work need n

Greek Letters

α_l	= index set of heat sources of level l
β_l	= index set of heat sinks for level l
Υ_l	= index set of steam needs for level l
ΔW_i	= loss of work at steam level i
η	= isentropic efficiency of a turbine
η_{lkn}	= mechanical efficiency of turbine n for expansion from pressure level l to pressure level k
η_m	= mechanical efficiency of a turbine
η_n	= efficiency of electric motor supplying work need n

ψ = availability per unit mass
 ω = angle between isentropic and real expansion lines in H-S diagram

Subscripts

ad = additional
 av = average
 b = auxiliary boiler
 m = mechanical
 s = isentropic process
 i = waste heat source
 j = heat sink
 k = process steam need
 n = work need

APPENDIX A: PROOF OF PROPOSITION 1

To prove the proposition we simply show that each of the three terms ΔW_i , ΔW_j , and ΔW_k is monotonically increasing.

1) Let $HS(P_i) = A$, $HS(P) = B$, $HS(P_L) = f_1(P_L)$ and $HW(P_L) = f_2(P_L)$ then

$$\Delta W_i = Q_i \frac{(f_1(P_L) - f_2(P_L))(A - B)}{(A - f_2(P_L))(B - f_2(P_L))}$$

Since f_2 is a monotonically increasing function of P_L and $f_2 < A$, $f_2 < B$, the denominator is a decreasing function of P_L . Clearly $A - B \geq 0$ since $P \leq P_i$. Therefore, the difference $(f_1(P_L) - f_2(P_L))$ is the key. For any fixed value of η and using the operating curve (Eq. 8) to determine T_L , steam tables or the Mollier diagram will show that the difference between the enthalpy of superheated steam at T_L and P_L and saturated water at P_L is a monotonically increasing function of P_L .

2) Using the definition of f_1 and f_2 given above, we have

$$\Delta W_j = Q_j \left\{ \left(\frac{HS(P)}{HS(P) - HW(P)} - \frac{HS(P_j)}{HS(P_j) - HW(P_j)} \right) + HS(P_L) \left(\frac{(f_1(P) - f_2(P)) - (f_1(P_j) - f_2(P_j))}{(f_1(P) - f_2(P))(f_1(P_j) - f_2(P_j))} \right) \right\}$$

Only the coefficient of $HS(P_L)$ is of concern since the other term is independent of P_L . Since $P_j \leq P$ and the difference $f_1(P) - f_2(P)$ is a monotonically increasing function in P , this coefficient is positive. Therefore, since $HS(P_L)$ is a monotonically increasing function of P_L , so is ΔW_j .

3) The term ΔW_k is independent of P_L . Thus, the proof is complete.

APPENDIX B: PROOF OF PROPOSITION 2

First, note that for a given set of problem data and fixed auxiliary boiler load Q_b , the header selection subproblem involves minimization of a separable, lower bounded function over a compact set of pressures. Consequently, for every Q_b existence of a finite minimum is guaranteed. Although the problem may exhibit multiple local minima, selection of the global minimum can be guaranteed by virtue of the enumerative nature of the dynamic programming procedure. Further, we assume on physical grounds that for any given Q_b , the minimum solution is unique. Hence, there exists a one-to-one mapping m which assigns to each Q_b a unique vector of pressures P . From energy balances, it is clear that for every network defined by P and any heat load Q there is a unique amount of work $g(Q, P)$ that can be generated by that network. By definition, $g(Q, P)$ will be maximum for given Q if $Q = Q_b$ and $P = m(Q_b)$, that is $W_{\max} = g(Q_b, m(Q_b))$. For any other Q , the following relationships can be developed:

Lemma 1

- Given any Q , $Q_1 > Q_2$ if and only if $g(Q_1, m(Q)) > g(Q_2, m(Q_1))$
- If $Q_1 \neq Q$, $g(Q_1, m(Q_1)) \geq g(Q_1, m(Q))$
- $Q_1 > Q$ if and only if $g(Q_1, m(Q_1)) > g(Q, m(Q))$

Proof. Statement (i) is an obvious consequence of the energy balance. Given any fixed network, more work can be obtained if and only if more heat is supplied. Statement (ii) is merely a re-statement of the definition of the maximum work attainable with a given Q . Statement (iii) is a consequence of (i) and the definition of maximum work.

Using Lemma 1 we next show that the Q_b adjustment procedure yields a monotonic sequence of work and Q values. Let W^* be the total work need which the network must satisfy and Q^* be the corresponding optimal boiler heat load.

Lemma 2

Assuming that the Q_b adjustment procedure is initiated with $Q = 0$, either for some n $Q_n = Q^*$ and $W_{n+1} = W^*$ or for all n the sequence of successive Q and W values will satisfy.

$$Q_1 > Q_2 > \dots > Q_{n-1} > Q_n > Q^*$$

$$W_2 > W_3 > \dots > W_n > W_{n+1} > W^*$$

Proof (by Induction). Since the process is initiated with $Q = 0$, we have $P_1 = m(0)$ and $W_1 = g(0, m(0))$. From the energy balance calculate Q_1 such that the required total work need W^* is met using the network $m(0)$. That is, $W^* = g(Q_1, m(0))$. Clearly, $W_1 < W^*$ because otherwise additional boiler duty is not required. By substitution we have, $g(0, m(0)) < g(Q_1, m(0))$. Therefore, from Lemma 1 (i), it follows that $Q_1 > 0$. Also, since by definition, $W^* = g(Q^*, m(Q^*))$, from Lemma 1 (iii), $Q^* > 0$.

Let $W_2 = g(Q_1, m(Q_1))$. Since, $Q_1 > 0$, Lemma 1 (ii) yields $W_2 = g(Q_1, m(Q_1)) \geq g(Q_1, m(0)) = W^*$

Clearly, if $W_1 = W^*$, $Q_1 = Q^*$ and the adjustment procedure terminates. Therefore, $W_2 > W^*$. In this case, however, using Lemma 1 (iii), $g(Q_1, m(Q_1)) > g(Q^*, m(Q^*))$ implies that, $Q_1 > Q^*$.

Next, select Q_2 such that $W^* = g(Q_2, m(Q_1))$. Since $W_2 > W^*$, we have $g(Q_1, m(Q_1)) > g(Q_2, m(Q_1))$ and thus from Lemma 1, (i), $Q_1 > Q_2$. Let $W_3 = g(Q_2, m(Q_2))$. From Lemma 1 (iii), $W_2 > W_3$ and from Lemma 1 (ii), $W_3 = g(Q_2, m(Q_2)) \geq g(Q_2, m(Q_1)) = W^*$. Again, $W_3 \neq W^*$ since otherwise the optimum has been obtained. Thus $W_3 > W^*$ and then from Lemma 1 (iii), $Q_1 > Q^*$. The above demonstration establishes that $Q_1 > Q_2 > Q^*$ and $W_2 > W_3 > W^*$.

Next, assume the hypothesis is true for k and show it holds for $k+1$. Choose Q_{k+1} such that $W^* = g(Q_{k+1}, m(Q_k))$. Since $W_{k+1} > W^*$, we have $g(Q_k, m(Q_k)) > g(Q_{k+1}, m(Q_k))$. From Lemma 1 (i), $Q_k > Q_{k+1}$. Let $W_{k+2} = g(Q_{k+1}, m(Q_{k+1}))$. Lemma 1 (iii) immediately yields that $W_{k+1} > W_{k+2}$ and, Lemma 1 (ii), that $W_{k+2} \geq W^*$.

If $W_{k+2} = W^*$, $Q_{k+1} = Q^*$. Otherwise $W_{k+2} > W^*$ which from Lemma 1 (iii) indicates that $Q_{k+1} > Q^*$. Lemma 2 is thus proven.

Since the sequence of W_n values generated by the adjustment procedure is monotonically decreasing with lower bound W^* , it follows that the sequence has a limit and that $\lim (W_n) = \inf (W_n)$ (Bartle, 1967).

Lemma 3: $\inf (W_n) = W^*$

Proof. Suppose the contrary is true, that is, $\inf (W_n) = \bar{W} \neq W^*$.

Clearly, $\bar{W} < W^*$ is impossible since $\bar{W} = \inf (W_n)$ and $W_n \geq W^*$ for all n . Therefore, $\bar{W} > W^*$. Given \bar{W} there must be a \bar{Q} such that, $\bar{W} = g(\bar{Q}, m(\bar{Q}))$. From Lemma 1 (iii), we have, $\bar{Q} > Q^*$. Next choose Q' so that $W^* = g(Q', m(\bar{Q}))$. Since $\bar{W} > W^*$, Lemma 1 (i) yields that $\bar{Q} > Q'$. Define $W = g(Q', m(Q'))$, then from Lemma 1 (ii), it follows that $W \geq W^*$. Also, from Lemma 1 (iii), we have $\bar{W} > W \geq W^*$ which contradicts the assumption that $\bar{W} = \inf (W_n)$. Proposition 2 is thus proven.

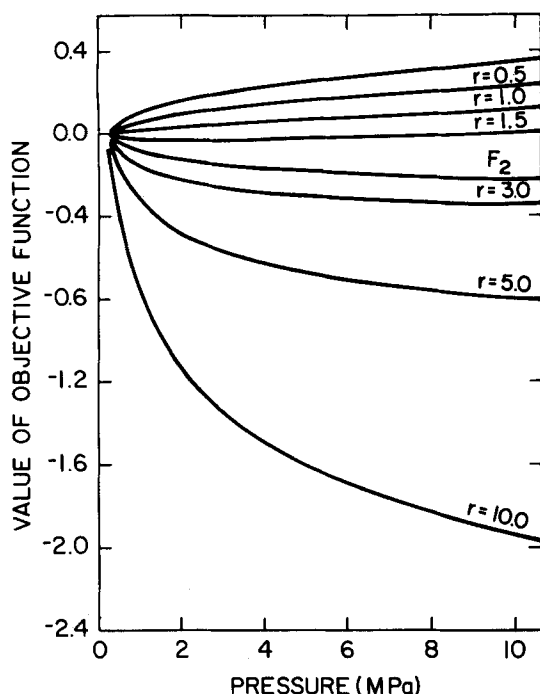


Figure 4. Change of objective function with pressure, boiler and heat exchanger case.

APPENDIX C: CHANGE OF OBJECTIVE FUNCTION WITH PRESSURE

At any given level l if $\alpha_l = \varphi$ and $\beta_l U \bar{T}_l \neq \varphi$ or if $\alpha_l \neq \varphi$, and $\beta_l U \bar{T}_l = \varphi$, clearly the optimum P_l will correspond to a candidate point. Therefore, the possibility of noncandidate point optimum pressure level occurs only if both $\alpha_l \neq \varphi$ and $\beta_l U \bar{T}_l \neq \varphi$. (The symbol φ denotes the empty set.)

Consider the case of a single source (steam boiler) and single sink (heating or process steam) at some level l . Specifically, suppose $\alpha_l = \{1\}$ and $\beta_l = \{2\}$. Then, for level l the objective function will consist of the two terms, $\Delta W_1(P_l) + \Delta W_2(P_l)$. The optimal value of P_l will lie in the region $P_2 \leq P_l \leq P_1$ since $\Delta W_1 = 0$ at $P_l = P_1$ and $\Delta W_1 > 0$ for $P_l > P_1$; $\Delta W_2 = 0$ at $P_l = P_2$ and $\Delta W_2 > 0$ for $P_l > P_2$. From Eqs. 1 and 2,

$$\Delta W_1(P) + \Delta W_2(P) = Q_1(HS(P_L) - HW(P_L)) \left(\frac{1}{HS(P) - HW(P_L)} - \frac{1}{HS(P_1) - HW(P_L)} \right) + Q_2 \left[\frac{HS(P) - HS(P_L)}{HS(P) - HW(P)} - \frac{HS(P_2) - HS(P_L)}{HS(P_2) - HW(P_2)} \right]$$

Since Q_1 , Q_2 , P_1 , P_2 , and P_L are constants, the variable part of the above equation is the portion

$$Q_1 \left(\frac{HS(P_L) - HS(P)}{HS(P) - HW(P_L)} \right) + Q_2 \left(\frac{HS(P) - HS(P_L)}{HS(P) - HW(P)} \right)$$

For simplicity this can be written as $rF_2(P) + F_1(P)$ where $r = Q_1/Q_2$.

Figure 4 gives plots of $F_1(P)$, $F_2(P)$ and $F_1(P) + rF_2(P)$ for various values of r all as functions of P . It is evident that F_1 is monotonically increasing and F_2 is monotonically decreasing with P . Note that if $r > 1.5$, $F_1 + rF_2$ is monotonically decreasing and, therefore, the minimum will occur at the upper candidate point P_1 . On the other hand, if $r \leq 1.0$, then the function $F_1 + rF_2$ is monotonically increasing and the minimum will occur at P_2 . Only for a limited range of ratios in the vicinity of 1.5 does the composite function exhibit an intermediate optimum. Moreover, for these cases the loss function is very flat and hence the value of the function at the true optimum differs by only a small relative amount from the objective function values at the bounds.

Although Figure 4 was plotted with $P_{\max} = 11.0$ MPa, $T_{\max} = 811$ K, $P_{\min} = 103$ kPa and $\eta = 0.7$, similar results can be obtained with other parameters values.

LITERATURE CITED

- Bartle, R. G., *Elements of Real Analysis*, John Wiley & Co., New York (1964).
- Bellman, R., *Dynamic Programming*, Princeton University Press, Princeton, NJ (1957).
- Gordon, E., M. H. Hasheni, R. D. Dodge, and J. LaRosa, "A Versatile Steam Balance Program," *Chem. Eng. Prog.*, **74**, 51 (July, 1978).
- Grossman, L. E., and J. Santibanez, "Applications of mixed-integer linear programming in process synthesis," Design Research Center, Carnegie-Mellon University, Pittsburgh, PA (Sept., 1979).
- deNevers, N., and J. D. Seader, "Mechanical Lost Work, Thermodynamic Lost Work and Thermodynamic Efficiencies of Processes," 86th AIChE National Meeting, Houston, Paper No. 30a (March, 1979).
- Nishida, N., G. Stephanopoulos, and A. Westerberg, "A Review of Process Synthesis," Carnegie-Mellon University, DRC Working Paper (March, 1980).
- Nishio, M., "Computer Aided Synthesis of Steam and Power Plants for Chemical Complexes," Ph.D. Dissertation, The University of Western Ontario, London, Canada (April, 1977).
- Nishio, M., J. Itoh, K. Shiroko, and T. Umeda, "A Thermodynamic Approach to Steam-Power Systems Design," *IEC Process Design and Development*, **19**, 308 (1980).
- Overturf, B. W., "A Physical Properties Package for Computer Aided Process Design," Purdue University, West Lafayette, IN (May, 1979).
- Spencer, R. C., K. C. Cotton, and C. N. Cannon, "A method for predicting the performance of steam turbine generators," ASME Papers WA, 62-WA-209, 201 (1962).
- Van Wylen, C. J., and R. E. Sonntag, *Fundamental of Classical Thermodynamics*, 239, Wiley, New York (1967).
- Wilkinson, C. R., "Computer Models for Heat Recovery and Utility Systems in Chemical Process Plants," M.S. Thesis, Purdue University, West Lafayette, IN (Aug. 1979).

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